Sequence and Series

1. **Definition: sequence:**

A succession of terms $a_1, a_2, a_3, \ldots, a_n$ formed according to some rule. ~~and~~ or, law.

Examples: $2, 4, 8, 14, 22, \cancel{25}$

$1, 4, 9, 16, 25, \ldots$

$-1, 1, -1, 1, \ldots$

$\frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \ldots$

A finite sequence has finite (limited) number of terms, as in first example above. And an infinite sequence has infinite or, an unlimited number of terms i.e there is no last term, as in rest three examples.

Series:

The indicated sum of terms of a sequence. In case of a finite sequence $a_1, a_2, a_3, \ldots, a_n$ and the corresponding series is $a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$. This series has a finite or limited number of terms and is called finite series.

(a) nth term is usually represented by t_n or, T_n.

4th term $\rightarrow t_4$ or, T_4

(b) sum of first n terms is usually denoted by S_n or s_n.

$S_5 \rightarrow$ Sum of first five terms

$S_5 = \cancel{S_1 + S_2 + S_3 + S_4 + S_5}$

$$S_5 = \underbrace{T_1 + T_2 + T_3 + T_4}_{S_4} + T_5$$

$\Rightarrow S_5 = S_4 + T_5$

$\Rightarrow T_5 = S_5 - S_4$

$\therefore$ for nth term:

$$\boxed{T_n = S_n - S_{n-1}}$$

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2. Arithmetic Progression (A.P.):

(I) The sequence in which difference of any two consecutive terms (directional) is constant, is called A.P.

(II) A general A.P can be given as:

$\underbrace{a,\ a+d}_{d},\ \underbrace{a+2d}_{d}, \dots, a+(n-1)d, \dots$

(III) The constant difference is called common difference.

common difference = 'd'

$\boxed{T_n - T_{n-1} = d}$ = constant

= Common difference

$T_{n-1} - T_n$ = − common difference

(IV) For A.P. → $\underset{T_1}{a},\ \underset{T_2}{a+d},\ \underset{T_3}{a+2d},\ \underset{T_4}{a+3d}, \dots$

1st term = a

common difference = d

nth term,

$$\boxed{T_n = a + (n-1)d}$$

$\boxed{\text{nth term} = \text{1st term} + (n-1)\ \text{common difference}}$

Illustration 1:-

For A.P. 4, 7, 10, ..., find 11th term.

Solution:

1st term = a = 4

common difference = d = 3

nth term:

$T_n = a + (n-1)d$

⇒ 11th term:

$T_{11} = a + (11-1)d$

$= 4 + 10(3)$

$= 34$

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Illustration 2:

Find 19th term of an A.P. $-8, -14, -20, \ldots$

Solution:

$a = -8$

$d = -14 - (-8) = -6$

$T_{19} = a + 18d$

$= -8 + 18(-6)$

$= -116$

Illustration 3!

For A.P. $29, 27, \ldots$ find first negative terms.

Solution:

$29, 27, \ldots$

$a = 29$

$d = 27 - 29 = -2$

let T_n be first negative term.

$T_n = a + (n-1)d$

$= 29 + (n-1)(-2)$

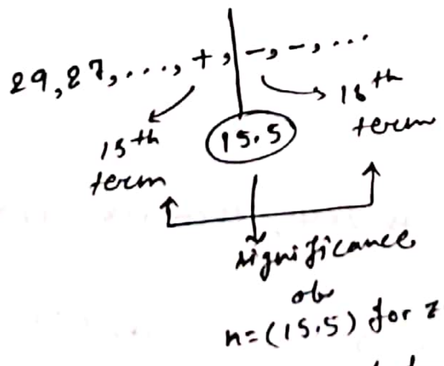
$= 29 - 2n + 2$

$= 31 - 2n \qquad \text{— (1)}$

let $T_n = 0$

$\Rightarrow 31 - 2n = 0$

$\Rightarrow n = \frac{31}{2} = 15.5$ ✗ (not a positive integer)
(that means zero will never come)



$T_{16} = 31 - 2 \cdot 16 \qquad$ (from eqn (1))

$= 31 - 32$

$= -1$

= value of 1st negative term

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Illustration 4:

let for a series, sum of 1st n terms is given by $S_n = n^2 - 4n$ then find 10th term.

Solution:

$$T_{10} = S_{10} - S_9$$
$$= (10^2 - 4\cdot 10) - (9^2 - 4\cdot 9)$$
$$= (100-40) - (81-36)$$
$$= 60 - 45$$
$$= 15$$

Illustration 5:

If $(x+1)$, $3x$ and $(4x+2)$ are first three terms of an A.P. then its 5th term is –

(A) 14 (B) 19 (C) 24 (D) 28

Solution:

$(x+1), 3x, (4x+2)$ are in A.P.

$$\Rightarrow 3x - (x+1) = (4x+2) - 3x$$
$$\Rightarrow 2x - 1 = x + 2$$
$$\Rightarrow \boxed{x = 3}$$

$$\therefore a = 4,\ d = 9 - 4 = 5$$
$$\Rightarrow T_5 = 4 + (5-1)5$$
$$= 4 + 20$$
$$= 24$$

Ans. (C)

Illustration 6:

If 19th term of a non-zero A.P. is zero, then its (49th term):(29th term) is:

(A) 4:1 (B) 1:3 (C) 3:1 (D) 2:1

(Jee-main 2019, 11th jan-II)

solution:

Let first term and common difference of A.P. be a and d respectively, then

$$t_n = a + (n-1)d$$

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$t_{19} = a + 18d = 0$

$\Rightarrow a = -18d \quad — (1)$

Now,

$$\frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d}$$

$$= \frac{-18d+48d}{-18d+28d}$$

$$= \frac{30d}{10d}$$

$$= 3$$

$t_{49} : t_{29} = 3:1$

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3. Sum of n terms for A.P. :

$a, a+d, a+2d, a+3d, \ldots, a+(n-1)d$

$$S_n = a + (a+d) + (a+2d) + (a+3d) + \cdots + (a+(n-1)d)$$
$$S_n = (a+(n-1)d) + (a+(n-2)d) + (a+(n-3)d) + \cdots + a$$

$$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + \cdots + (2a+(n-1)d)$$
$$= n(2a+(n-1)d)$$

$$\Rightarrow \boxed{S_n = \frac{n}{2}\{2a+(n-1)d\}} \Rightarrow \text{Memorize}$$

$$S_n = \frac{n}{2}\{\underbrace{a}_{T_1} + \underbrace{a+(n-1)d}_{T_n}\}$$
$$= \frac{n}{2}(T_1+T_n)$$
$$= \left(\frac{\text{no. of terms}}{2}\right)(1^{st}\text{ term} + \text{last term})$$

Note:

(i) $a, a+d, a+2d, a+3d, \ldots$

$$T_n = a+(n-1)d$$
$$T_r = a+(r-1)d$$
$$= d\cdot r + (a-d)$$
$$T_r = dr + (a-d)$$
$$\equiv \text{linear expression of } 'r'$$

Also,

$T_r = \alpha r + \beta$ (let for any sequence)

$T_{r-1} = \alpha(r-1) + \beta$

$T_r - T_{r-1} = \alpha = \text{constant}$ (difference of two consecutive terms is constant and hence in A.P.)

For any A.P., its r^{th} term will be linear expression of 'r' (and vice-versa), also coefficient of r will be common difference.

Examples are: $\underbrace{3,7}_{4}\underbrace{,11}_{4}\underbrace{,15}_{4},\ldots$

$$\boxed{T_n = 4n-1}$$

$7, 11, 15, 19, \ldots$

$$\boxed{T_n = 4n+3}$$

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$15, 13, 11, \ldots$
$\underbrace{}_{-2}\underbrace{}_{-2}$

$$\boxed{T_n = -2n + 17}$$

<ii> $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{n}{2}\{nd + (2a - d)\}$

$= \left(\frac{d}{2}\right)n^2 + \left(\frac{2a-d}{2}\right)n$

$$\Rightarrow \boxed{S_n = \left(\frac{d}{2}\right)n^2 + \left(\frac{2a-d}{2}\right)n}$$

For an A.P., sum of 1st n terms will be quadratic expression of 'n' with constant term as zero and vice-versa is also true.

Illustration 7 :

Find sum of 1st 20 terms of an A.P. 31, 27, 23, ...,

Solution:

$a = 31$

$d = -4$

$n = 20$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_{20} = \frac{20}{2}\{2.31 + (20-1)(-4)\}$

$= 10(62 - 76)$

$= -140$

Illustration 8 :

Find maximum possible sum of A.P. 40, 37, 34, ... n terms

Solution:

$a = 40$

$d = -3$

$40, 37, 34, \ldots, \_, \_, \ldots$

Maximum possible sum

1st negative term

$T_n = a + (n-1)d$

$= 40 + (n-1)(-3)$

$= 43 - 3n$

$T_n = 0$

$\Rightarrow 43 - 3n = 0 \Rightarrow n = \frac{43}{3} = 14.\ldots$th.

Significance: in this series, there'll be no term as zero.

$40, 37, 34, \ldots, 4, \;|\; \_, \_, \ldots$

15th term

14th term

$\frac{1}{2}$, 1st negative term

(14.33) term

$\therefore S_{max} = S_{14} = \frac{14}{2}\{2.40 + (14-1)(-3)\}$

$= 7(80-39)$

$= 7 \times 41$

$= 287$

$$\boxed{\begin{array}{l}\text{Required maximum sum} = 287 \\ \text{corresponding } n = 14\end{array}}$$

Note:

If $n = 15$ in place of $n = 14.33$ then T_{16} would have been 1st negative term and T_{14} would have been last positive term. Also, in this case $S_{14} = S_{15}$ $(\because T_{15} = 0)$, i.e. corresponding $n = 14, 15$.

Illustration 9:

Sum of 1st p terms of an A.P. is q and sum of 1st q terms of an A.P. is p then find summation of first $(p+q)$ terms.

Solution:

$S_p = q \Rightarrow \frac{p}{2}\{2a + (p-1)d\} = q$ — (i)

$S_q = p \Rightarrow \frac{q}{2}\{2a + (q-1)d\} = p$ — (ii)

Now, from equation (i) - (ii), we get;

$2a\left(\frac{p}{2} - \frac{q}{2}\right) + \frac{d}{2}\left(p(p-1) - q(q-1)\right) = q - p$

$\Rightarrow a(p-q) + \frac{d}{2}\left((p^2-p) - (q^2-q)\right) = -(p-q)$

$\Rightarrow a(p-q) + \frac{d}{2}\left((p+q)(p-q) - (p-q)\right) = -(p-q)$

$\Rightarrow a + (p+q-1)\frac{d}{2} = -1 \quad (\because p \neq q)$

$\Rightarrow 2a + (p+q-1)d = -2$

Now,

$S_{p+q} = \frac{p+q}{2}\left(2a + (p+q-1)d\right)$

$= \left(\frac{p+q}{2}\right)(-2)$

$= -(p+q)$

$$\boxed{\therefore S_{p+q} = -(p+q)}$$

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Illustration 10:

Let for a series, sum of 1st n terms is $S_n = n^2+4n$ T.P.T. it is an A.P.

Solution:

$S_n = n^2+4n$

$S_{n-1} = (n-1)^2+4(n-1)$

$S_n - S_{n-1} = (2n-1)+4$

$= 2n+3$

$\because T_n = S_n - S_{n-1}$

$\Rightarrow T_n = 2n+3$

$T_{n-1} = 2(n-1)+3$

$\underbrace{T_n - T_{n-1}}_{\text{difference of two consecutive terms}} = 2 = \text{constant}$

$\Rightarrow$ Given series is in A.P.

Illustration 11:

For an A.P. $T_{10} = \frac{1}{20}$; $T_{20} = \frac{1}{10}$, find sum of first 200 terms.

(A) $201\frac{1}{2}$ (B) $101\frac{1}{2}$ (C) $301\frac{1}{2}$ (D) $100\frac{1}{2}$

(Jee-main 2020, 8th Jan-D)

Solution:

$T_{10} = \frac{1}{20}$

$\Rightarrow a+9d = \frac{1}{20}$ — (1)

$T_{20} = \frac{1}{10}$

$\Rightarrow a+19d = \frac{1}{10}$ — (11)

Now, from equation (11) − (1), we get;

$10d = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$

$\Rightarrow d = \frac{1}{200}$

Putting the value of d in equation (1), we have;

$a+9\left(\frac{1}{200}\right) = \frac{1}{20}$

$\Rightarrow a = \frac{1}{200}$

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$\therefore S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right]$

$= \frac{201}{2}$

$= 100\frac{1}{2}$

$\Rightarrow \boxed{S_{200} = 100\frac{1}{2}}$

Illustration 12:

In an A.P., if $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$ then show that $S_{mn} = \frac{1}{2}(mn+1)$. (AIEEE 1982) (similar question)

$(m \neq n)$ ($t_{mn} = ?$)

solution:

let 1st term be a and C.D. be d.

$T_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n}$ — (I)

$T_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m}$ — (II)

from equation (I) - (II); we get,

$(m-n)d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$

$\Rightarrow d = \frac{1}{mn}$

Now, from equation (I),

$a + (m-1)\frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a + \cancel{\frac{1}{n}} - \frac{1}{mn} = \cancel{\frac{1}{n}}$

$\therefore a = \frac{1}{mn}$

$\therefore S_{mn} = \frac{mn}{2}(2a + (mn-1)d)$

$= \frac{mn}{2}\left(\frac{2}{mn} + (mn-1)\frac{1}{mn}\right)$

$= \frac{\cancel{mn}}{2} \cdot \frac{mn+1}{\cancel{mn}}$

$= \frac{mn+1}{2}$

$\Rightarrow \boxed{S_{mn} = \frac{mn+1}{2}}$

Note!

Procedure based Questions : Be 100% sure to solve.

IQ based Questions!

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Illustration 13:

The sum of first n terms of two A.Ps. are in ratio $\frac{7n+1}{4n+27}$. Find the ratio of their 11th term.

Solution:

let A.P.$_1$: $a, a+d, a+2d, \ldots$

A.P.$_2$: $A, A+D, A+2D, \ldots$

for A.P.$_1$,

$$s_n = \frac{n}{2}\{2a+(n-1)d\}$$

for A.P.$_2$,

$$S_n = \frac{n}{2}\{2A+(n-1)D\}$$

Given:

$$\frac{s_n}{S_n} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{\frac{n}{2}\{2a+(n-1)d\}}{\frac{n}{2}\{2A+(n-1)D\}} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27} \quad \text{— (I)}$$

Required:

$$\frac{t_{11}}{T_{11}} = \frac{a+10d}{A+10D} \quad \text{— (II)}$$

Comparing equations (I) and (II), we get;

$$\frac{n-1}{2} = 10$$

$$\Rightarrow n = 21$$

Putting the value of n in equation (I),

$$\frac{a+10d}{A+10D} = \frac{7\cdot 21+1}{4\cdot 21+27} = \frac{148}{111} = \frac{4}{3}$$

$$\boxed{\text{Required ratio} = 4:3}$$

Note:

If $\frac{t_n}{T_n}$ were asked.

$$\frac{t_n}{T_n} = \frac{a+(n-1)d}{A+(n-1)D}$$

; line of thinking: think interms of m then substitute m by n.

$n \to 2m-1$ both sides & $\frac{t_m}{T_m} = \frac{14m-6}{8m+23}$

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Do Yourself - 2:

(i) Write down the sequence whose nth term is:

(a) $\frac{2^n}{n}$ (b) $\frac{3+(-1)^n}{3^n}$

(ii) For an A.P. show that $t_m + t_{2n+m} = 2t_{m+n}$

(iii) If sum of first p terms of an A.P. is same as sum of 1st q terms then find S_{p+q}.

(iv) 4, 10, 16, ..., 88 is an A.P. then find

(a) No. of terms

(b) Sum of all terms

(c) How many terms should be taken such that summation exceeds 120?

(v) The first term of an A.P. is 5, the last term is 45, and the sum is 400. Find the number of terms and the common difference.

(vi) let -297, -294, -291, ... is an A.P. then find

(a) Common difference

(b) nth term and sum of n terms

(c) The value of n such that S_n is least.

(d) Which term is first non-negative term.

(e) Which term is first positive term.

(f) How many terms should be taken such that sum of terms is 18?

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4. Properties of A.P.

(a) To show that three numbers a, b, c are in A.P. we can show that,

$$b-a = c-b$$

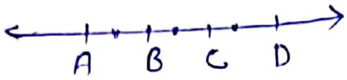
or,

$$2b = a+c$$

or,

$$(b-a)-(c-b)=0$$

(b) Numbers in A.P. will be equidistant on number line, if they are real.



(c) $a,\ a+d,\ a+2d, \dots$ A.P.

(i) $\xrightarrow{+k}$ $a+k,\ a+k+d,\ a+k+2d, \dots$ also in A.P. with same C.D.

$\xrightarrow{\cdot k}$ $a.k,\ ka+kd,\ ka+2kd, \dots$ also in A.P. with C.D. $= kd$ or, C.D. will be k times that of original.

(ii) $a,\ a+d,\ a+2d, \dots$

$A,\ A+D,\ A+2D, \dots$

$\xrightarrow{+}$ $a+A,\ a+A+d+D,\ a+A+2d+2D, \dots$ also in A.P. with C.D. as sum of CDs of given APs.

(d) If odd number of terms are to be taken in A.P. then take them symmetrically around

$$\dots, a-d, a, a+d, \dots$$

Three numbers in A.P. : $a-d,\ a,\ a+d$

Five numbers in A.P. : $a-2d,\ a-d,\ a,\ a+d,\ a+2d$

(e) If even number of terms are to be taken then take them symmetrically around,

$$\dots, \underbrace{a-d,\ a+d}_{2d}, \dots$$

Four terms in A.P. : $a-3d,\ a-d,\ a+d,\ a+3d$

Six terms in A.P. : $a-5d,\ a-3d,\ a-d,\ a+d,\ a+3d,\ a+5d$

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(f) The common difference can be zero, positive or, negative.

(g) K^{th} term from the last $= (n-K+1)^{th}$ term from begining
(if total no. of terms $= n$).

(h) The sum of the two terms of an A.P. equidistant from begining and end is constant and equal to first and last terms. $\Rightarrow T_K + T_{n-K+1} = \text{constant}$
$= a + l$

(i) Any term of an A.P. (except the first) is equal to half the sum of terms which are equidistant from it.

$$a_n = \frac{a_{n-K} + a_{n+K}}{2}, \quad K < n$$

For $K=1$;

$$a_n = \frac{a_{n-1} + a_{n+1}}{2}$$

For $K=2$;

$$a_n = \frac{a_{n-2} + a_{n+2}}{2}$$

and so on.

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Illustration 14:

Four numbers are in A.P. If their sum is 20 and the sum of their squares is 120, then the middle terms are -

(A) 2, 4 (B) 4, 6 (C) 6, 8 (D) 8, 10

Solution:

Let the numbers are $a-3d, a-d, a+d, a+3d$

Given:

$$(a-3d)+(a-d)+(a+d)+(a+3d)=20$$

$$\Rightarrow 4a=20$$

$$\Rightarrow a=5$$

Also,

$$(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=120$$

$$\Rightarrow 4a^2+20d^2=120$$

$$\Rightarrow 4\times 5^2+20\times d^2=120$$

$$\Rightarrow d^2=1$$

$$\Rightarrow d=\pm 1$$

Hence numbers are:

2, 4, 6, 8 (if $d=1$)

8, 6, 4, 2 (if $d=-1$)

Illustration 15:

If a, b, c are in A.P. then prove that $b+c, c+a, a+b$ are also in A.P.

Solution:

Given: a, b, c are in A.P.

T.P. $b+c, c+a, a+b$ are in A.P.

Now,

a, b, c are in A.P.

Adding $-(a+b+c)$ in each term, we get;

$a-a-b-c, b-a-b-c, c-a-b-c$ are in A.P.

$\Rightarrow -b-c, -a-c, -a-b$ are in A.P.

Multiplying by (-1) in each term, we get;

$b+c, c+a, a+b$ are in A.P.

Hence proved

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Illustration 16 :

Given: $a_1, a_2, a_3, \ldots, a_n$ are in A.P. Then prove that

$$\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \cdots + \frac{1}{a_n a_1} = \frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\cdots+\frac{1}{a_n}\right]$$

solution :

We know that,

$$a_1 + a_n = a_2 + a_{n-1}$$
$$= a_3 + a_{n-2} = \cdots$$

$$\text{LHS} = \frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1 a_n} + \frac{a_1+a_n}{a_2 a_{n-1}} + \frac{a_1+a_n}{a_3 a_{n-2}} + \cdots + \frac{a_1+a_n}{a_n a_1}\right]$$

$$= \frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1 a_n} + \frac{a_2+a_{n-1}}{a_2 a_{n-1}} + \cdots + \frac{a_1+a_n}{a_1 a_n}\right]$$

$$= \frac{1}{a_1+a_n}\left[\left(\frac{1}{a_1}+\frac{1}{a_n}\right) + \left(\frac{1}{a_2}+\frac{1}{a_{n-1}}\right) + \cdots + \left(\frac{1}{a_n}+\frac{1}{a_1}\right)\right]$$

$$= \frac{1}{a_1+a_n}\left(\frac{2}{a_1} + \frac{2}{a_n} + \frac{2}{a_2} + \frac{2}{a_{n-1}} + \cdots\right)$$

$$= \frac{2}{a_1+a_n}\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_n}\right)$$

$$= \text{RHS}$$

Do Yourself - 2 :

<i> Find the sum of first 24 terms of the A.P. $a_1, a_2, a_3, \ldots, a_{24}$ if it is known that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

<ii> Find the number of terms common to the two A.P.'s
3, 7, 11, 15, ..., 407 and
2, 9, 16, 23, ..., 709.

<iii> If $a_1, a_2, a_3, \ldots, a_n$ are in A.P. where $a_i > 0$ for all i, show that:

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

<iv> If a^2, b^2, c^2 are in A.P. T.P.T. $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

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5. Geometric Progression:

(I) The sequence in which ratio of any two consecutive terms is constant is called Geometric progression (G.P.).

(II) This constant ratio is called common ratio.
(The preceeding term should be in denominator)

(III) a, b, c, d are in G.P.

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{1}{C.R.}$$

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = C.R.$$

(III) A G.P. can be given as

$a, ar, ar^2, \ldots$
$T_1 \quad T_2 \quad T_3$

1st term $= a$

Common ratio $= r$

kth term $= ar^{k-1}$

$$\boxed{n\text{th term} = ar^{n-1}}$$

(IV)

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$
$$rS_n = \quad ar + ar^2 + \ldots + ar^{n-1} + ar^n$$

$$(1-r)S_n = a - ar^n$$
$$= a(1-r^n)$$

$$\boxed{\therefore S_n = \frac{a(1-r^n)}{(1-r)}} \quad \text{if } r \neq 1$$

$$\Rightarrow \boxed{S_n = \frac{a(r^n-1)}{(r-1)}} = \frac{\text{1st term}\left((C.R.)^{\text{no. of terms}} - 1\right)}{(C.R. - 1)}$$

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(V) Infinite G.P. :

Let $0 < |r| < 1$ then,

$a, ar, ar^2, ar^3, \ldots \infty$

$a + ar + ar^2 + ar^3 + \ldots \infty$

$$= \frac{a(r^n - 1)}{(r-1)}, \; n \to \infty$$

$$= \lim_{n \to \infty} \frac{a(r^n - 1)}{(r-1)}, \quad r^n \to 0$$

$$= \frac{a(-1)}{(r-1)}$$

$$= \frac{a}{1-r}$$

$$\boxed{\therefore \; a + ar + ar^2 + \ldots \infty = \frac{a}{1-r}}, \; 0 < |r| < 1$$

$$\Rightarrow \boxed{S_\infty = \frac{a}{1-r}}, \; 0 < |r| < 1$$

6. Properties of G.P. :

(a) To show that a, b, c are in G.P. then S.T.

either, $\boxed{\frac{b}{a} = \frac{c}{b}}$

or, $\boxed{b^2 = ac}$

(b) If odd number of terms are to be taken in G.P. then take them symmetrically around

3 terms: $\frac{a}{r}, a, ar \to$ C.R. $= r$

5 terms: $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2 \to$ C.R. $= r$

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(c) If even number of terms are to be taken in G.P. then take them symmetrically around

4 terms: $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3 \rightarrow$ C.R. $= r^2$

6 terms: $\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}, ar, ar^3, ar^5 \rightarrow$ C.R. $= r^2$

(d) (i) $a, ar, ar^2, ar^3, \ldots$ G.P.

$\cdot K$ $\rightarrow Ka, Kar, Kar^2, \ldots$ also in G.P. with same C.R.

(ii) $a, ar, ar^2, ar^3, \ldots$ G.P. 1

$A, AR, AR^2, AR^3, \ldots$ G.P. 2

$\rightarrow aA, aArR, aAr^2R^2, \ldots$ G.P. with C.R. $= rR$

= product of C.R.s of given GPs

(e) If in a G.P., the product of two terms which are equidistant from the first term and the last term is constant ~~then~~ and is equal to the product of first and last term.

$$\Rightarrow \boxed{T_K \cdot T_{n-K+1} = \text{constant} = a.l}$$

(f) In a G.P., $T_r^2 = T_{r-K} \cdot T_{r+K}$, $K<r$, $r \neq 1$

(h) If the terms of a G.P. be raised to the same power, then the resulting sequence is also a G.P.

(i) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(j) If $a_1, a_2, a_3, \ldots, a_n$ are a G.P. of positive terms then $\log a_1, \log a_2, \log a_3, \ldots, \log a_n$ is an A.P. and vice-versa.

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